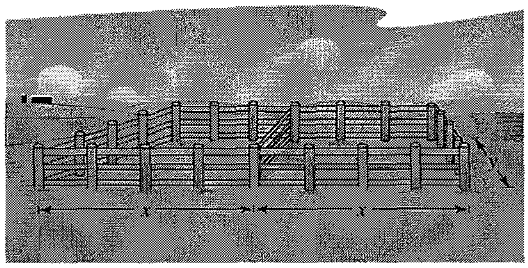
All work must be shown to be awarded full credit.
Provide exact solutions to all problems, unless otherwise stated. A scientific calculator is allowed.

Student Name: KEY **ID:** ______

Instructor: Mundy-Castle

Exam Score: ______

1. A rancher has 720 feet of fencing with which to enclose two adjacent rectangular corrals (see figure). What dimensions should be used so that the enclosed area will be a maximum?



Maximize area

$A = 2xy$

$A = 2x\left(240 - \frac{4}{3}x\right)$

$= 480x - \frac{8}{3}x^2$

$A'(x) = 480 - \frac{16}{3}x$

$-\frac{16}{3}x + 480 = 0$

$\frac{16}{3}x = 480$

$x = 90$ ft

$y = 240 - \frac{4}{3}(90) = 120$ ft

Secondary equation

$4x + 3y = 720$

$3y = 720 - 4x$

$y = 240 - \frac{4}{3}x$

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2. Determine whether the Mean Value Theorem can be applied to f on the closed interval $[a, b]$. If the Mean Value Theorem can be applied, find all values of c in the open interval (a, b) such that $f'(c) = \frac{f(b)-f(a)}{b-a}$.

$$f(x) = \sqrt{x} - 2x, \quad [0, 4]$$

MVT can be applied: f is continuous on $[0,4]$, differentiable on $(0,4)$

$f(x) = x^{1/2} - 2x, \quad f'(x) = \frac{1}{2}x^{-1/2} - 2 = \frac{1}{2\sqrt{x}} - 2$

$$\frac{f(4)-f(0)}{4-0} = \frac{-6-0}{4} = -\frac{3}{2}$$

$$\frac{1}{2\sqrt{x}} - 2 = -\frac{3}{2} \rightarrow \frac{1}{2\sqrt{x}} = \frac{1}{2}$$

$$\sqrt{x} = 1 \rightarrow x = 1$$

$$\boxed{c = 1}$$

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3. Let $f(x) = \frac{4x}{x^2+1}$ be a function defined on its domain.

a) Find the intervals on which f is increasing and/or decreasing.

b) Classify the relative extrema of f.

$$f'(x) = \frac{4(x^2+1) - 4x(2x)}{(x^2+1)^2} = \frac{4x^2+4-8x^2}{(x^2+1)^2}$$

$$= \frac{-4x^2+4}{(x^2+1)^2} \quad 6$$

$$-4x^2+4=0 \rightarrow 4x^2=4 \rightarrow x^2=1$$

2 each

$x=1, \; x=-1 \leftarrow$ critical #'s

a)

	$(-\infty,-1)$ 2	$(-1,1)$ 2	$(1,\infty)$ 2
T.V.	$x=-2$	$x=0$	$x=2$
sign of f'	$-$	$+$	$-$
conclusion	$\downarrow$	$\uparrow$	$\downarrow$

decreasing on $(-\infty,-1)\cup(1,\infty)$

increasing on $(-1,1)$

b) $f(-1)=-2$; relative min. at $(-1,-2)$ 2

$f(1)=2$; relative max. at $(1,2)$ 2

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4. Consider $f(x) = 2x^2(1 - x^2)$.

a) Determine the inflection points of f.

b) State the intervals of concavity for f.

$f(x) = 2x^2 - 2x^4$

$f'(x) = 4x - 8x^3$

$f''(x) = 4 - 24x^2$

$4 - 24x^2 = 0$

$24x^2 = 4$

$x^2 = \frac{1}{6}$

$x = \pm\sqrt{1/6} = \pm\frac{\sqrt{6}}{6}$

possible inflection pts.

	$\left(-\infty, -\frac{\sqrt{6}}{6}\right)$	$\left(-\frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{6}\right)$	$\left(\frac{\sqrt{6}}{6}, \infty\right)$
T.V	$x = -1$	$x = 0$	$x = 1$
Sign of f''	$-$	$+$	$-$
Conclusion	C.D.	C.U.	C.D.

a) $f\left(-\frac{\sqrt{6}}{6}\right) = \frac{5}{18}$ $f\left(\frac{\sqrt{6}}{6}\right) = \frac{5}{18}$

Inflection pts. at $\left(-\frac{\sqrt{6}}{6}, \frac{5}{18}\right)$ and $\left(\frac{\sqrt{6}}{6}, \frac{5}{18}\right)$

b) concave down on $\left(-\infty, -\frac{\sqrt{6}}{6}\right) \cup \left(\frac{\sqrt{6}}{6}, \infty\right)$

concave up on $\left(-\frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{6}\right)$

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5. Find the limit algebraically.

$$\lim_{x\to\infty} \frac{-3x+1}{\sqrt{x^2+x}}$$

$$\lim_{x\to\infty} \frac{(-3x+1)\left(\frac{1}{x}\right)}{\sqrt{x^2+x}\left(\frac{1}{x}\right)} = \lim_{x\to\infty} \frac{-3+\frac{1}{x}}{\frac{\sqrt{x^2+x}}{x}}$$

$$= \lim_{x\to\infty} \frac{-3+\frac{1}{x}}{\frac{\sqrt{x^2+x}}{\sqrt{x^2}}} = \lim_{x\to\infty} \frac{-3+\frac{1}{x}}{\sqrt{\frac{x^2+x}{x^2}}}$$

$$= \lim_{x\to\infty} \frac{-3+\cancel{\frac{1}{x}}^{\to 0}}{\sqrt{1+\cancel{\frac{1}{x}}_{\to 0}}} = \frac{-3}{\sqrt{1}} = -3$$